Toward getting finite results from $\mathcal{N} = 4$ SYM with α'-corrections

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Abstract

We take our first step toward getting finite results from the α'-corrected D=4 N=4 SYM theory with emphasis on the field theory techniques. Starting with the classical action of the N=4 SYM with the leading α'-corrections, we examine new divergence at one loop due to the presence of the α'-terms. The new vertices do not introduce additional divergence to the propagators or to the three-point correlators. However they do introduce new divergence, e.g., to the scalar four-point function which should be canceled by extra counter-terms. We expect that the counter-terms will appear in the 1PI effective action that is obtained by considering the string annulus diagram. We work out the structure of the divergence and comment on an application to the anomalous dimension of the SYM operators in the context of AdS/CFT.

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1 Introduction

In the recent developments of string theory the $D = 4$ $\mathcal{N} = 4$ SYM theory has played a much important role. The prominent example is AdS/CFT correspondence where $\mathcal{N} = 4$ SYM theory is employed to study aspects of IIB supergravity/string theory on $\mathrm{AdS}_5 \times \mathrm{S}_5$. The $D = 4$ $\mathcal{N} = 4$ theory approximates an open superstring attached to a set of D3-branes (see, e.g., [1] for a review) : the results obtained by a full-fledged string computation will reduce to those of the SYM in the $\alpha' \to 0$ limit. Since it is a leading order approximation it may be worth studying a theory that better approximates the open string than the pure SYM. We will consider a classical action that is obtained by considering the open string disc diagram, the α'-corrected SYM. The action was obtained in ten dimensions [2, 3, 4, 5, 6, 7, 8]. We keep the leading α'-correction terms which come at α'^2-order[1] and reduce it to four dimensions.

While $\mathcal{N} = 4, D = 4$ SYM theory is a super-renormalizable theory the status drastically changes once one adds the corrections from the string theory since those correction terms are power-counting non-renormalizable. The presence of the new vertices generates additional divergence. In general even with a non-renormalizable field theory one can consider an order-by-order renormalization, but then the theory suffers from the loss of predictive power. This would not be the case with the action of our starting point, the SYM with α'-corrections, since it comes from the string theory. As well known open superstring yields finite results to various scattering amplitudes, which are obtained via the world-sheet technique. Therefore it may be worth seeing how the finiteness results in the field theory context where divergence occurs. The divergence would have to be cancelled by counter-terms. Here we study the structures of the divergence and possible forms of the counter-terms. It will be interesting to confirm (or disconfirm) that the open string annulus diagram indeed implies the presence of such terms. We leave the check to the future string theory based computation [12].

The remainder of the paper is organized as follows. In sec2, we consider the SYM plus the α'^2-results that are obtained in the literature [5, 6, 7]. These are ten dimensional results: we carry out dimensional reduction to four dimensions. Out of the terms that result we record only the terms that are relevant for our computation. More complete expression is presented in Appendix B. With the dimensional regularization we examine, at one-loop and α'^2 order, various divergence. We note that the new vertices do not introduce any new divergence to the propagators due to an identity concerning the scaleless integrals in the dimensional regularization. We move to new three point correction graphs, which vanish as well. With the three point correlators it is the color index structure that makes them vanish. Non-vanishing divergence appears with four-point functions. We take the example of the scalar four-point functions and

[1] In the literature (e.g.,[9, 10, 11]) a few higher orders were obtained as well for the bosonic sector.

work out the divergent parts of the integrals. We then look into the possible forms of the counter-terms. We illustrate this with an example. Section 3 has discussions of issues that are related to the current computation. We also comment on future directions. In Appendix A, we present our notations and conventions for the SYM, and list the Z-factors of the wave-function renormalization.

2 New divergence from sringy vertices

The N=4 action with the α'-corrections is quoted in the appendices along with our conventions. Below we consider, at one-loop and α'^2-order, new graphs to two-, three- and the four- point functions that are introduced by the stringy vertices. In the case of the four point function we only consider the scalar external lines. The new graphs of two and three point function vanish, but as for the four point graphs one has non-vanishing results. We analyze their structure and discuss the counter terms that remove the divergence.

2.1 propagators and three-point functions

The stringy vertices produce new graphs of radiative corrections to the propagtors. We present a few of them in Fig.1 below. They (and all the other propagator corrections at α'^2-order) vanish due to an identity concerning the scaleless integrals in the dimensional regularization (see, e.g.,[13, 14]),

$$\int d^d q \ (q^2)^\beta = 0 \tag{1}$$

where β is an arbitrary number.

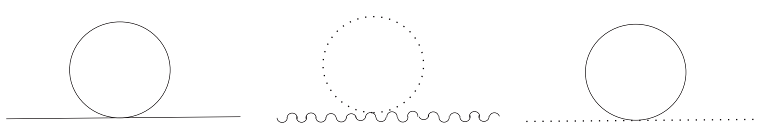

Figure 1: Examples of the new graphs for the propagators

The one loop corrections to the three point correlators also vanish: some of them for the same reason as the propagator corrections. There are other graphs that do not contain the scaleless integrals. They have two vertices, one from the pure SYM and the other from the α'^2-vertices as illustrated in Fig.2. These graphs vanish because of their color structure: they all come with

$$\sim f^{def} \ \mathrm{Str}(T^e T^f \cdots) = 0 \tag{2}$$

where T's are the SU(N) group generators in the adjoint representation, ${T^b}_{ac} = i f^{abc}$

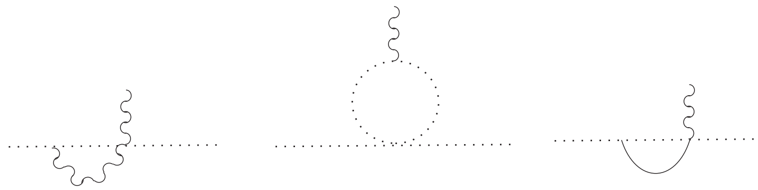

Figure 2: Examples of the new graphs for the three point function

2.2 four-scalar vertices

At one-loop, there are altogether five graphs shown in Fig.3. In each graph one vertex comes from the α'^2-terms and the other(s) from the pure SYM part. The stringy vertices are presented in Appendix B. All the graphs contain the common factor of

$$(2\pi\alpha')^2 g_{YM}^8 f^{mea} f^{mfb}\, \mathrm{Str}(T^e T^f T^c T^d)\, \frac{(2\pi)^4 \delta(\sum_{k=1}^4 p_k)}{p_1^2 p_2^2 p_3^2 p_4^2} \frac{\Gamma(2-\omega)}{(4\pi)^2} \tag{3}$$

where $\omega \equiv D/2$. Our conventions are explained in Appendix A. We summarize our

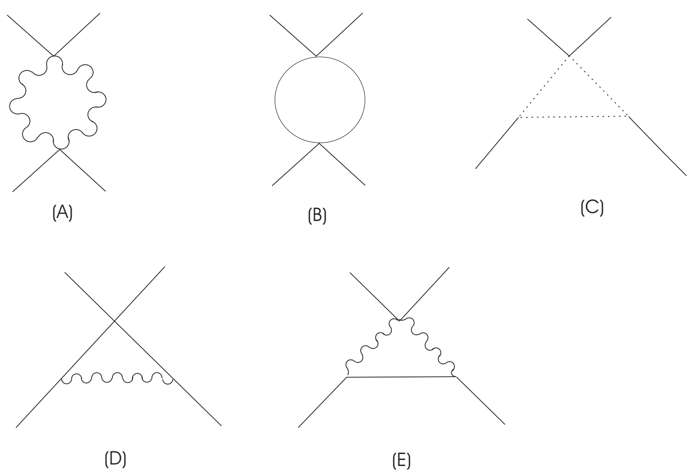

Figure 3: New graphs for the scalar four point function

results as follows. One of the two vertices in graph (A), with the other being one of the standard SYM vertices, comes from

$$(2\pi\alpha')^2 \;\; \mathrm{Str}\left[-\frac{1}{8} F_{\mu\nu} F^{\mu\nu} D_\rho \phi_k D^\rho \phi^k - \frac{1}{2} D_\nu \phi_i F_{\nu\rho} F^{\rho\sigma} D^\sigma \phi^i\right] \tag{4}$$

Here and below only the regular partial derivative part of the covariant derivative will contribute. The sum of these two contributions is given by[2]

$$< \phi_i^a(x_1)\phi_j^b(x_2)\phi_k^c(x_3)\phi_l^d(x_4) >_{(A)} \Rightarrow \delta_{ij}\delta_{kl}\left[\frac{1}{24}(p_1+p_2)^2(p_3\cdot p_4) - \frac{1}{6}\;\; (p_1+p_2)\cdot p_3\;(p_1+p_2)\cdot p_4\right]$$

[2] "$\Rightarrow$" indicates the fact that only the divergent parts have been recorded. The results are given in the momentum space obtained by Fourier transformation. Here and below the perm stands for the

$$+\text{perm} \qquad (5)$$

The stringy vertex in graph (B) come from

$$(2\pi\alpha')^2 \ \text{Str}\left[-\frac{1}{8}D_\mu\phi_j D^\mu\phi^j D_\nu\phi_k D^\nu\phi^k + \frac{1}{4}D_\nu\phi_i D^\nu\phi^k D_\sigma\phi_k D^\sigma\phi^i\right] \qquad (6)$$

which yields

$$\begin{aligned} < \phi_i^a(x_1)\phi_j^b(x_2)\phi_k^c(x_3)\phi_l^d(x_4) >_{(B)} \Rightarrow \quad & \delta_{ij}\delta_{kl}\left[\frac{1}{6}(p_1+p_2)^2\ p_3\cdot p_4 - \frac{5}{12}\ (p_1+p_2)\cdot p_3\ (p_1+p_2)\cdot p_4\right] \\ & +\delta_{ik}\delta_{jl}\left[\frac{1}{4}\ (p_1+p_2)^2\ p_3\cdot p_4\right] + \text{perm} \qquad (7) \end{aligned}$$

For graph (C) the relevant terms are

$$(2\pi\alpha')^2 \ \text{Str}\left[\frac{1}{4}\bar\psi\Gamma_\mu D_\nu\psi D^\mu\phi^i D^\nu\phi_i - \frac{1}{4}\bar\psi\Gamma_{\mu nk}D_\sigma\psi D^\mu\phi^n D^\sigma\phi\right] \qquad (8)$$

The result for graph (c) as it comes out of the computation is

$$\begin{aligned} < \phi_i^a(x_1)\phi_j^b(x_2)\phi_k^c(x_3)\phi_l^d(x_4) >_{(C)} \Rightarrow \quad & \delta_{ij}\delta_{kl}[-\frac{1}{3}(p_1+p_2)^2\ p_3\cdot p_4 - \frac{2}{3}\ (p_1+p_2)\cdot p_3\ (p_1+p_2)\cdot p_4 \\ & -(p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_3)(p_2\cdot p_4) - (p_1\cdot p_4)(p_2\cdot p_3) \\ & -2(p_2\cdot p_3)(p_2\cdot p_4)] \\ & +\delta_{ik}\delta_{jl}\left[2(p_1\cdot p_3)(p_2\cdot p_4) - 2(p_1\cdot p_4)(p_2\cdot p_3)\right] + \text{perm} \end{aligned}$$

It can be simplified by utilizing the SO(6)- and color- index structures:

$$\begin{aligned} < \phi_i^a(x_1)\phi_j^b(x_2)\phi_k^c(x_3)\phi_l^d(x_4) >_{(C)} \Rightarrow \quad & \delta_{ij}\delta_{kl}[-\frac{1}{3}(p_1+p_2)^2\ p_3\cdot p_4 - \frac{2}{3}\ (p_1+p_2)\cdot p_3\ (p_1+p_2)\cdot p_4 \\ & -(p_1\cdot p_2)(p_3\cdot p_4) - (p_1\cdot p_3)(p_1\cdot p_4) - (p_2\cdot p_3)(p_2\cdot p_4)] \\ & +\delta_{ik}\delta_{jl}\left[2(p_1\cdot p_3)(p_2\cdot p_4) - 2(p_1\cdot p_4)(p_2\cdot p_3)\right] + \text{perm} \end{aligned} \qquad (9)$$

The stringy vertex for graph (D) is the same as that of (B). One gets the following result,

$$\begin{aligned} < \phi_i^a(x_1)\phi_j^b(x_2)\phi_k^c(x_3)\phi_l^d(x_4) >_{(D)} \Rightarrow \quad & \delta_{ij}\delta_{kl}\left[-\frac{5}{8}(p_1\cdot p_2)(p_3\cdot p_4) - \frac{1}{12}\ p_1^2(p_3\cdot p_4) - \frac{1}{8}p_2^2(p_3\cdot p_4)\right. \\ & \left. +\frac{1}{12}(p_1\cdot p_3)(p_1\cdot p_4) + \frac{1}{4}(p_1\cdot p_4)(p_2\cdot p_3)\right] \end{aligned}$$

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terms that are obtained by permutations of

$$\{(p_1,a,i),(p_2,b,j),(p_3,c,k),(p_4,d,l)\}$$

$$
\begin{aligned}
&+\delta_{ik}\delta_{jl}\Big[-\frac{1}{4}(p_1\cdot p_3)(p_2\cdot p_4)+\frac{1}{4}(p_1\cdot p_4)(p_2\cdot p_3)\\
&\qquad+\frac{1}{2}p_2^2(p_3\cdot p_4)+\frac{1}{4}p_1^2(p_3\cdot p_4)+\frac{9}{4}(p_1\cdot p_2)(p_3\cdot p_4)\Big]\\
&+\text{perm}
\end{aligned}
$$

which can be rewritten, by utilizing the SO(6)- and color- index structures, as

$$
\begin{aligned}
< \phi_i^a(x_1)\phi_j^b(x_2)\phi_k^c(x_3)\phi_l^d(x_4) >_{(D)}\Rightarrow \quad & \delta_{ij}\delta_{kl}\Big[-\frac{5}{12}(p_1\cdot p_2)(p_3\cdot p_4)-\frac{5}{48}\,(p_1+p_2)^2(p_3\cdot p_4)\\
&\quad+\frac{1}{12}(p_1\cdot p_3)(p_1\cdot p_4)+\frac{1}{8}(p_1\cdot p_3)(p_2\cdot p_4)\\
&\quad+\frac{1}{8}(p_1\cdot p_4)(p_2\cdot p_3)\Big]\\
&+\delta_{ik}\delta_{jl}\Big[-\frac{1}{4}(p_1\cdot p_3)(p_2\cdot p_4)+\frac{1}{4}(p_1\cdot p_4)(p_2\cdot p_3)\\
&\quad+\frac{1}{2}p_2^2(p_3\cdot p_4)+\frac{1}{4}p_1^2(p_3\cdot p_4)\\
&\quad+\frac{9}{4}(p_1\cdot p_2)(p_3\cdot p_4)\Big]+\text{perm}
\end{aligned}
\tag{10}
$$

Finally the graph (E), whose stringy vertex is the same as that of (A), yields vanishing result:

$$
< \phi_i^a(x_1)\phi_j^b(x_2)\phi_k^c(x_3)\phi_l^d(x_4) >_{(E)}\Rightarrow \quad 0 \tag{11}
$$

Summing up (5)-(10) one gets

$$
\begin{aligned}
< \phi_i^a(x_1)\phi_j^b(x_2)\phi_k^c(x_3)\phi_l^d(x_4) >_{\text{total}}\Rightarrow \quad & \delta_{ij}\delta_{kl}\Big[-\frac{11}{48}\,(p_1+p_2)^2(p_3\cdot p_4)-\frac{5}{4}\,\ (p_2{\cdot}p_3)(\,p_2{\cdot}p_4)\\
&\quad-\frac{5}{12}(p_1\cdot p_2)(p_3\cdot p_4)-\frac{7}{6}(p_1\cdot p_3)(p_1\cdot p_4)\\
&\quad-\frac{9}{8}(p_1\cdot p_3)(p_2{\cdot}_4\,)-\frac{9}{8}(p_1\cdot p_4)(p_2\cdot p_3)\Big]\\
&+\delta_{ik}\delta_{jl}\Big[+\frac{7}{4}(p_1\cdot p_3)(p_2\cdot p_4)-\frac{7}{4}(p_1\cdot p_4)(p_2\cdot p_3)\\
&\quad+\frac{3}{4}p_2^2(p_3\cdot p_4)+\frac{1}{2}p_1^2(p_3\cdot p_4)\\
&\quad+\frac{11}{4}(p_1\cdot p_2)(p_3\cdot p_4)\Big]+\text{perm}
\end{aligned}
\tag{12}
$$

The counter-terms that remove the divergence can readily be obtained. We illustrate this with $\delta_{ik}\delta_{jl}\ \frac{11}{4}(p_1\cdot p_2)(p_3\cdot p_4)$-term in (12). Including the common factor (3) it is

$$
(2\pi\alpha')^2 g_{YM}^8 f^{mea}f^{mfb}\ \mathrm{Str}(T^eT^fT^cT^d)\ \frac{(2\pi)^4\delta(\sum_{k=1}^4 p_k)}{p_1^2p_2^2p_3^2p_4^2}\frac{\Gamma(2-\omega)}{(4\pi)^2}\delta_{ik}\delta_{jl}\ \frac{11}{4}(p_1\cdot p_3)(p_2\cdot p_4)
\tag{13}
$$

It can be removed by adding the following counter-term in the action

$$-\frac{11}{4}(2\pi\alpha')^2 g_{YM}^2 f^{mep} f^{mfq}\,\mathrm{Str}(T^e T^f T^c T^d)\,\frac{\Gamma(2-\omega)}{(4\pi)^2}\,\partial_\mu\phi_m^p\partial_\mu\phi_n^q\partial_\nu\phi_m^g\partial_\nu\phi_n^h \qquad (14)$$

The counter-terms for other parts of the divergence can be similarly determined.

3 Discussions and Future Directions

One of the reasons why the present computation may be useful is the fact that a D-brane is a stringy object: it will take the full open string theory for a complete description of the object. The methods of the description of a D-brane are at the heart of the AdS/CFT. The relevance of the open string in the context of AdS/CFT was discussed e.g., in [15, 16, 17].[3] The leading approximation of the open superstring is the SYM theory. Although simple and useful it does not contain the effects of the massive open string modes. Therefore it may be meaningful to try to accommodate them. There are two ways to do that. First one may turn to the world-sheet description for various scattering amplitudes. At a given loop order, it will include the complete effects of the massive modes. Less inclusive but still advantageous in other aspects is the regular field theory approach. Efficient to include the massive modes, the world-sheet theory does not have the same status as a regular field theory since string field theory is less developed although there has been some progress [20, 21]. Furthermore, unlike the abelian case where the effective action can be obtained in a closed form (see, e.g., [22] for a relatively recent discussion), in the non-abelian case one must consider four-point, five-point, etc, separately, and deduce the field theory action from the results. It may be useful for that purpose to know the possible forms of the field theory counter-terms in advance through an analysis such as the present one. In other words, the string-based technique and the field theory technique may be mutually guiding.[4].

We comment on two potential applications of our results. In the literature, there have been pieces of evidence [25, 26, 27] that the perturbative quantum corrections of pure SYM theory can be mapped to the terms in the DBI action in the $\mathrm{AdS}_5\times\mathrm{S}^5$ curved background. (Related discussions can be found in [28, 29].) Once we complete the check of the counter-terms through the string analysis we will be in a position to see how they would modify the story. Presumably they would not change the big picture but only some details such as the field redefinition introduced in [27]. The other application is that one may investigate whether/how the α'-terms correct the anomalous dimensions of the SYM operators that appear in the context of AdS/CFT [30].

[3] Related discussions may be found in [18, 19].

[4] Related discussions for the pure SYM case can be found in [23, 24]

We end with a few side remarks. One way to interpret the results of [25, 26, 27] is that putting the action in the curved background amounts to having the 1PI effective action, Γ. In other words, although one starts out with the SYM (or open string) in a flat space the theory completes itself in the curved target space(in the sense of a non-linear sigma type model). The advantage of having the 1PI action handy is that one only computes the tree graphs since the action already contains all the quantum corrections. Therefore to compute certain physical quantities one can either start with the flat space action and include the quantum corrections, or alternatively use the 1PI action, which would be equivalent to using the action in the curve space[5], and compute the tree graphs. However, for certain purposes such as mechanically finding the SYM operators that are dual to the supergravity modes [31, 17, 32] or implementing the duality at a lagrangian-to-lagrangian level [16][6] it seems to take the action in the curved background from the beginning.

Acknowledgments

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[5]The effective action in [27] was obtained by S-dualizing the SYM one-loop effive action, hence would contain not only the perturbative quantum effect but also the non-perturbative effect although there is some subtlety as expressed in [27]. Therefore it is more than a 1PI action since typically a 1PI action refers only to the parts that are obtained by perturbative techniques.

[6]The computational techniques are curved space generalization of those of [33, 34].

Appendix A: Notations and Conventions

$\mathcal{N}=4$ SYM action with the leading string correction is given by

$$\mathcal{L}=\mathcal{L}_{SYM}+\mathcal{L}_c \tag{A.1}$$

with

$$\begin{aligned}\mathcal{L}_{SYM} \;=\; & \Big[-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}-\frac{1}{2}\left(\partial_\mu\phi^a_i+f^{abc}A^b_\mu\phi^c_i\right)^2-\frac{1}{2}\bar{\psi}^a\Gamma^\mu\left(\partial_\mu\psi^a+f^{abc}A^b_\mu\psi^c\right) \\ & -\frac{1}{2}\,f^{abc}\,\bar{\psi}^a\Gamma^i\phi^b_i\;\psi^c-\frac{1}{4}\sum_{i,j}f^{abc}f^{ade}\phi^b_i\phi^c_j\phi^d_i\phi^e_j-\frac{1}{2}\partial_\mu\omega^*_a\left(\partial^\mu\omega_a+f^{abc}A^\mu_b\omega_c\right)\Big]\end{aligned} \tag{A.2}$$

where ψ is a thirty two component Mayorana-Weyl spinor with four dimensional space-time dependence. The conjugation is is defined by

$$\bar{\psi}\equiv\psi^\dagger i\Gamma^0 \tag{A.3}$$

The α'^2-order terms in $\mathcal{L}_c$ (which is the leading correction) are given in Appendix B. To take into account the fact that ψ is a Mayorana-Weyl spinor one uses the following relation [35] at the end of the trace algebra,

$$\mathrm{tr}\;\Gamma^\mu\Gamma^\nu=16\delta^{\mu\nu} \tag{A.4}$$

The Z-factors of the wave-function renormalization are as follows:

$$Z_\phi=1+\frac{\lambda}{8\pi^2}\Gamma(2-w)\qquad Z_\psi=1+\frac{4\lambda}{16\pi^2}\Gamma(2-w)\qquad Z_A=1+\frac{\lambda}{8\pi^2}\Gamma(2-w) \tag{A.5}$$

The first two Z-factors are given, e.g., in [35].

Appendix B: Dimensional reduction of the leading α'-corrections

In $D=10$ Minkowski space the $\mathcal{N}=1$ SYM action with leading string corrections [2, 3, 5, 6, 7] is

$$\begin{aligned}\mathcal{L}_{\alpha'^2,D=10} \;=\; & \mathrm{Str}\;\;(2\pi)^2\alpha'^2\Big[\frac{1}{8}F^{MN}F_{NP}F^{PQ}F_{QM}-\frac{1}{32}\left(F^{MN}F_{MN}\right)^2 \\ & -\frac{1}{4}\,\bar{\psi}\Gamma_M D_N\psi\;F^{MP}F_P{}^N+\frac{1}{8}\,\bar{\psi}\Gamma_{MNP}D_Q\psi\,F^{MN}F^{PQ}+\frac{1}{24}\,\bar{\psi}\Gamma^M D^N\psi\,\bar{\psi}\Gamma_M D_N\psi \\ & +\frac{7}{480}F_{MN}\,\bar{\psi}\Gamma^{MNP}\psi\,\{\bar{\psi},\Gamma_P\psi\}-\frac{\tilde{\alpha}^2}{2880}F_{MN}\,\bar{\psi}\Gamma_{PQR}\psi\,\{\bar{\psi},\Gamma^{MNPQR}\psi\}\Big]\end{aligned} \tag{B.1}$$

where Str denotes the symmetrized trace on color indices which are suppressed,

$$\mathrm{Str}\, A_1 A_2 \cdots A_n = \frac{1}{n!}\mathrm{tr}\,(A_1 A_2 \cdots A_n + \text{all permutations}) \tag{B.2}$$

Keeping the terms with up to two fermion fields, the dimensionally reduced action is as follows:

$$\begin{aligned}
\frac{\mathcal{L}_{\alpha'^2,D=4}}{(2\pi)^2\alpha'^2} = \quad & -\frac{1}{32}\Big(F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + [\phi_i,\phi_j][\phi^i,\phi^j][\phi_k,\phi_l][\phi^k,\phi^l] + 4D_\mu\phi_j D^\mu\phi^j D_\nu\phi_k D^\nu\phi^k \\
& \quad -2F_{\mu\nu}F^{\mu\nu}[\phi_i,\phi_j][\phi^i,\phi^j] + 4F_{\mu\nu}F^{\mu\nu}D_\rho\phi_k D^\rho\phi^k - 4[\phi_i,\phi_j][\phi^i,\phi^j]D_\mu\phi_k D^\mu\phi^k\Big) \\
& +\frac{1}{8}\Big(F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu} + [\phi_i,\phi_j][\phi^j,\phi^k][\phi_k,\phi_l][\phi^l,\phi^i] - 4D_\nu\phi_i F_{\nu\rho}F^{\rho\sigma}D^\sigma\phi^i \\
& \quad +2D_\nu\phi_i D^\nu\phi^k D_\sigma\phi_k D^\sigma\phi^i + 4iD_\nu\phi_i F^{\nu\rho}D_\rho\phi_l[\phi^l,\phi^i] + 4D_\nu\phi_i D^\nu\phi^k[\phi_k,\phi_l][\phi^l,\phi^i]\Big) \\
& -\frac{1}{4}\Big(\bar\psi\Gamma_\mu D_\nu\psi F^{\mu\rho}F_\rho{}^\nu - \bar\psi\Gamma_\mu D_\nu\psi D^\mu\phi^i D^\nu\phi_i - i\bar\psi\Gamma_\mu[\phi_i,\psi]F^{\mu\nu}D_\nu\phi^i \\
& \quad +\bar\psi\Gamma_i D_\mu\psi F^{\mu\nu}D_\nu\phi^i + \bar\psi\Gamma_\mu[\phi_i,\psi]D^\mu\phi^j[\phi^i,\phi_j] - i\bar\psi\Gamma_i D_\mu\psi D^\mu\phi^j[\phi^i,\phi_j] \\
& \quad +i\bar\psi\Gamma_i[\phi_j,\psi]D^\mu\phi^i D_\mu\phi_j + i\bar\psi\Gamma_i[\phi_j,\psi][\phi^i,\phi^k][\phi_k,\phi^j]\Big) \\
& +\frac{1}{8}\Big(\bar\psi\Gamma_{\mu\nu\rho}D_\sigma\psi F^{\mu\nu}F^{\rho\sigma} - i\bar\psi\Gamma_{\mu\nu\rho}[\phi_l,\psi]F^{\mu\nu}D^\rho\phi^l - \bar\psi\Gamma_{\mu\nu k}D_\sigma\psi F^{\mu\nu}D^\sigma\phi^k \\
& \quad -2\bar\psi\Gamma_{\mu\rho n}D_\sigma\psi D^\mu\phi^n F^{\rho\sigma} - \bar\psi\Gamma_{\mu\nu k}[\phi_l,\psi]F^{\mu\nu}[\phi^k,\phi^l] + 2i\bar\psi\Gamma_{\mu\rho n}[\phi_l,\psi]D^\mu\phi^n D^\rho\phi^l \\
& \quad -2\bar\psi\Gamma_{\mu nk}D_\sigma\psi D^\mu\phi^n D^\sigma\phi^k - 2\bar\psi\Gamma_{\mu nk}[\phi_l,\psi]D^\mu\phi^n[\phi^k,\phi^l] \\
& \quad -\bar\psi\Gamma_{mn\rho}[\phi_l,\psi][\phi^m,\phi^n]D^\rho\phi^l - i\bar\psi\Gamma_{mn\rho}D_\sigma\psi[\phi^m,\phi^n]F^{\rho\sigma} \\
& \quad +i\bar\psi\Gamma_{mnk}D_\sigma\psi[\phi^m,\phi^n]D^\sigma\phi^k + i\bar\psi\Gamma_{mnk}[\phi_l,\psi][\phi^m,\phi^n][\phi^k,\phi^l]\Big) + \cdots
\end{aligned} \tag{B.3}$$

where ψ is a thirty two component Mayorana-Weyl spinor with four dimensional space-time dependence.

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